Exercise 62

Prove, without graphing, that the graph of the function has at least two x-intercepts in the specified interval.

$$y = x^2 - 3 + 1/x, \quad (0, 2)$$

Solution

The function $f(x) = x^2 - 3 + 1/x$ is the sum of two functions, $g(x) = x^2 - 3$ and $h(x) = 1/x$, which are both continuous on their respective domains by Theorem 7. And by Theorem 4, $f(x)$ is continuous wherever both $g(x)$ and $h(x)$ are. Evaluate the function at several values of x in the interval of interest.

$$f(0.25) \approx 1.06$$

$$f(0.5) = -0.75$$

$$f(0.75) \approx -1.10$$

$$f(1) = -1$$

$$f(1.25) \approx -0.638$$

$$f(1.5) \approx -0.0833$$

$$f(1.75) \approx 0.634$$

$$f(2) = 1.5$$

$f(x)$ is continuous on the closed interval $[0.25, 0.5]$, and $N = 0$ lies between $f(0.25)$ and $f(0.5)$. By the Intermediate Value Theorem, then, there exists an x-intercept within $0.25 < x < 0.5$. Also, $f(x)$ is continuous on the closed interval $[1.5, 1.75]$, and $N = 0$ lies between $f(1.5)$ and $f(1.75)$. By the Intermediate Value Theorem, then, there exists another x-intercept within $1.5 < x < 1.75$. Therefore, there are at least two x-intercepts in the interval $(0, 2)$—more can potentially be found by evaluating $f(x)$ at even more values of x.